



## Grid Generation for Fusion Applications

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### ► To cite this version:

Hervé Guillard, Jalal Lakhili, Adrien Loseille, Alexis Loyer, Ahmed Ratnani. Grid Generation for Fusion Applications. EFTC 2017 - 17th European Fusion Theory Conference, Oct 2017, Athens, Greece. pp.1. hal-01644309

**HAL Id: hal-01644309**

**<https://hal.inria.fr/hal-01644309>**

Submitted on 22 Nov 2017

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# Grid Generation for Fusion Applications

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## Magnetized plasma

- An extremely anisotropic medium :
  - Transport coefficients in the parallel (to the magnetic field) and perpendicular directions differ by several order of magnitude :  $k_{//}/k_{\perp} \sim 10^9$
  - Dynamics is completely different in the parallel and perpendicular direction : incompressible in the perp direction, compressible (and supersonic in the SOL) in the parallel direction : plasma slides along the magnetic field lines
- Plasma - wall interaction (Bohm's boundary condition) : Plasma enter into the wall at supersonic velocity :

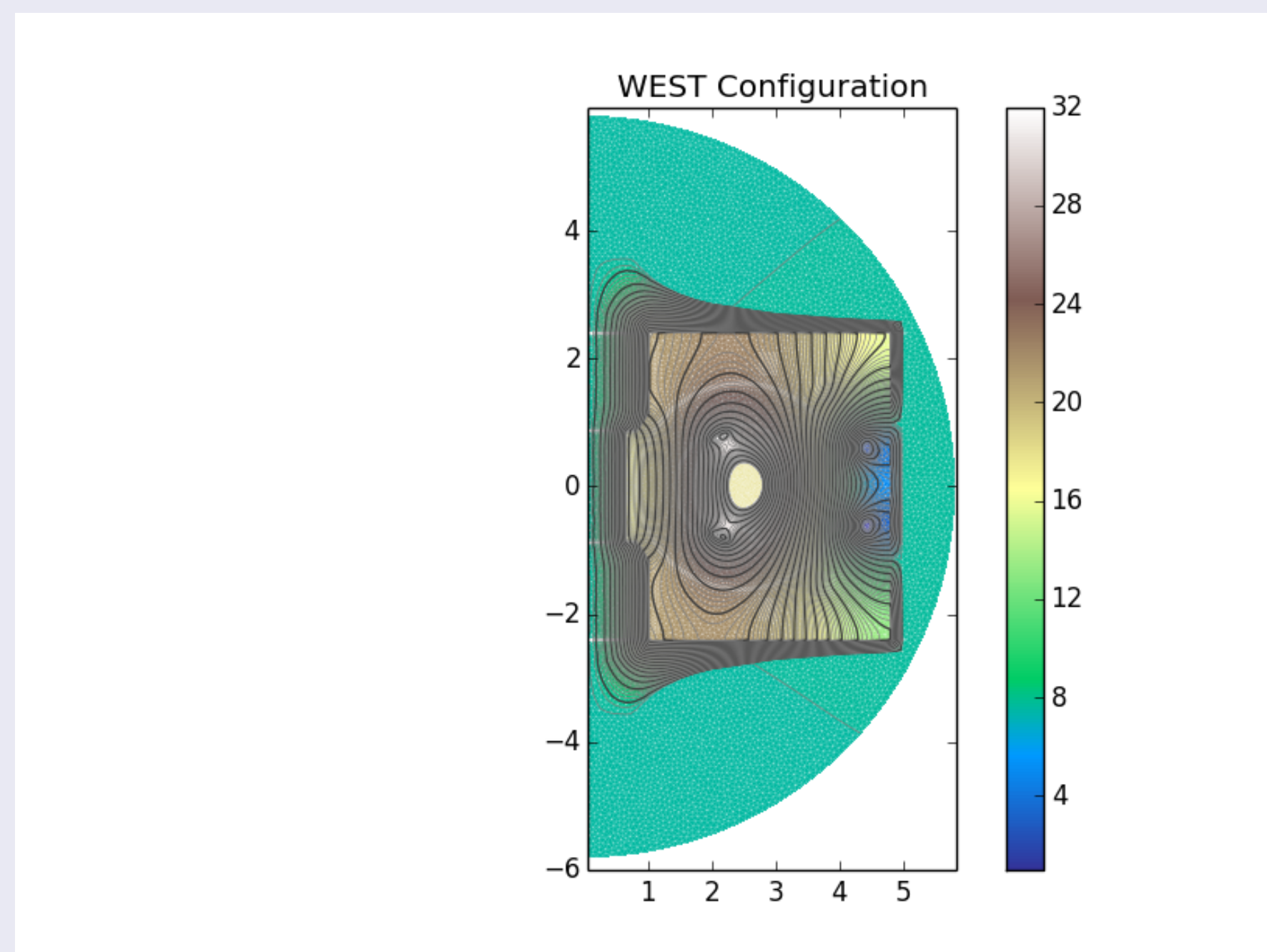
**In magnetized plasma, plasma flows along the magnetic field lines**

## Meshes adapted to magnetic flux surfaces

## Grad-Shafranov solvers

For realistic geometries, magnetic field has to be computed by numerical Grad-Shafranov solvers

$$-\nabla \left( \frac{1}{\mu r} \nabla \psi \right) = \begin{cases} \mathbf{J}(\mathbf{x}, \psi) & \text{plasma} \\ \mathbf{J}(\text{voltage}, \partial_r \psi) & \text{coils} \\ 0 & \text{elsewhere} \end{cases}$$



## Block-structured grid

- Block-structured meshes :
  - Identify the sub-domains (blocks-patches)  $\Omega_n$
  - Construct the mappings between logical grid  $[0, 1] \times [0, 1]$  and physical  $\Omega_n$
- How many blocks and definition of the patches ?

This is a **segmentation problem** :

Given a domain  $\Omega$  : Find a number  $n$  and  $n$  patches  $\Omega_n$  such that :

$$\Omega = \cup \Omega_n \text{ with } \overset{\circ}{\Omega}_n \cap \overset{\circ}{\Omega}_m = \emptyset$$

and there exists a one to one mapping between  $\Omega_n$  and the the unit square :

$$\exists \phi_n : [0, 1] \times [0, 1] \leftrightarrow \Omega_n$$

- for flux surface aligned grid, the segmentation problem can be solved by **Morse theory**

## Segmentation Problem and Morse Function

$f$  is a Morse function if all its critical points are regular.

### Critical points

Let  $\mathcal{C}^r$  be the space of  $r \geq 2$  differentiable scalar field defined on  $\Omega$ .  $\mathbf{p} \in \Omega$  is a critical point of  $f$  if  $\nabla f(\mathbf{p}) = 0$ .

### Regular Critical points

A critical point is regular if the Hessian of  $f$  at  $\mathbf{p}$  is invertible.

### In 2-D

the only possible critical points of a Morse function are :

Maxima index = 2 Minima index = 0

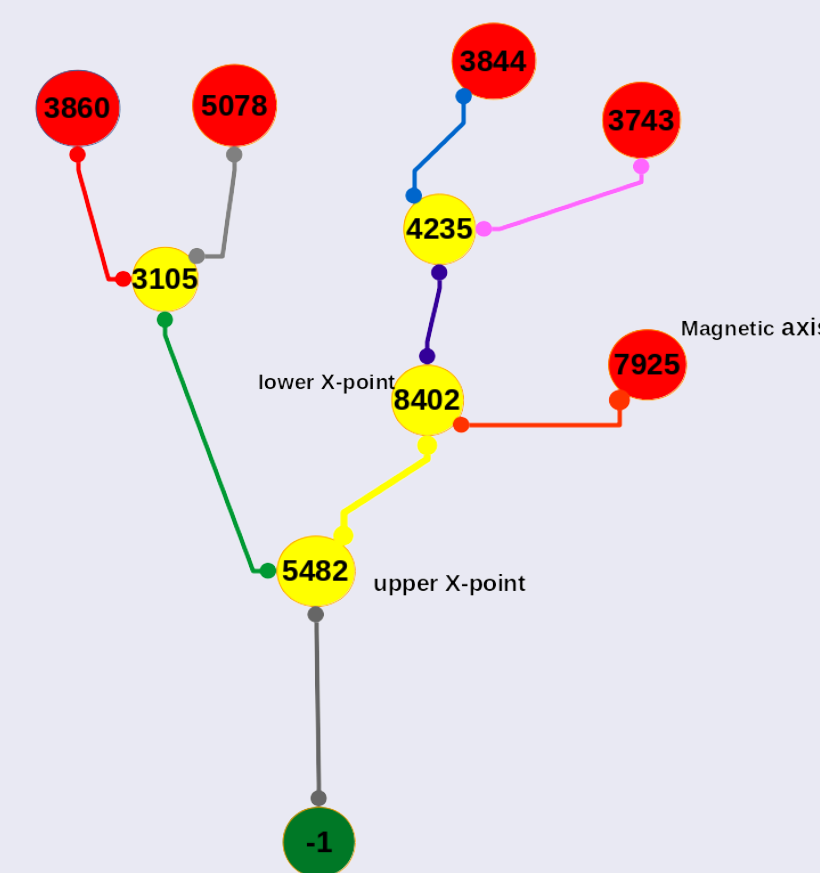
Saddles index = 1

**Topology of a Morse function** : The topological set of the iso-contours of  $f$  consists of connected components that are either

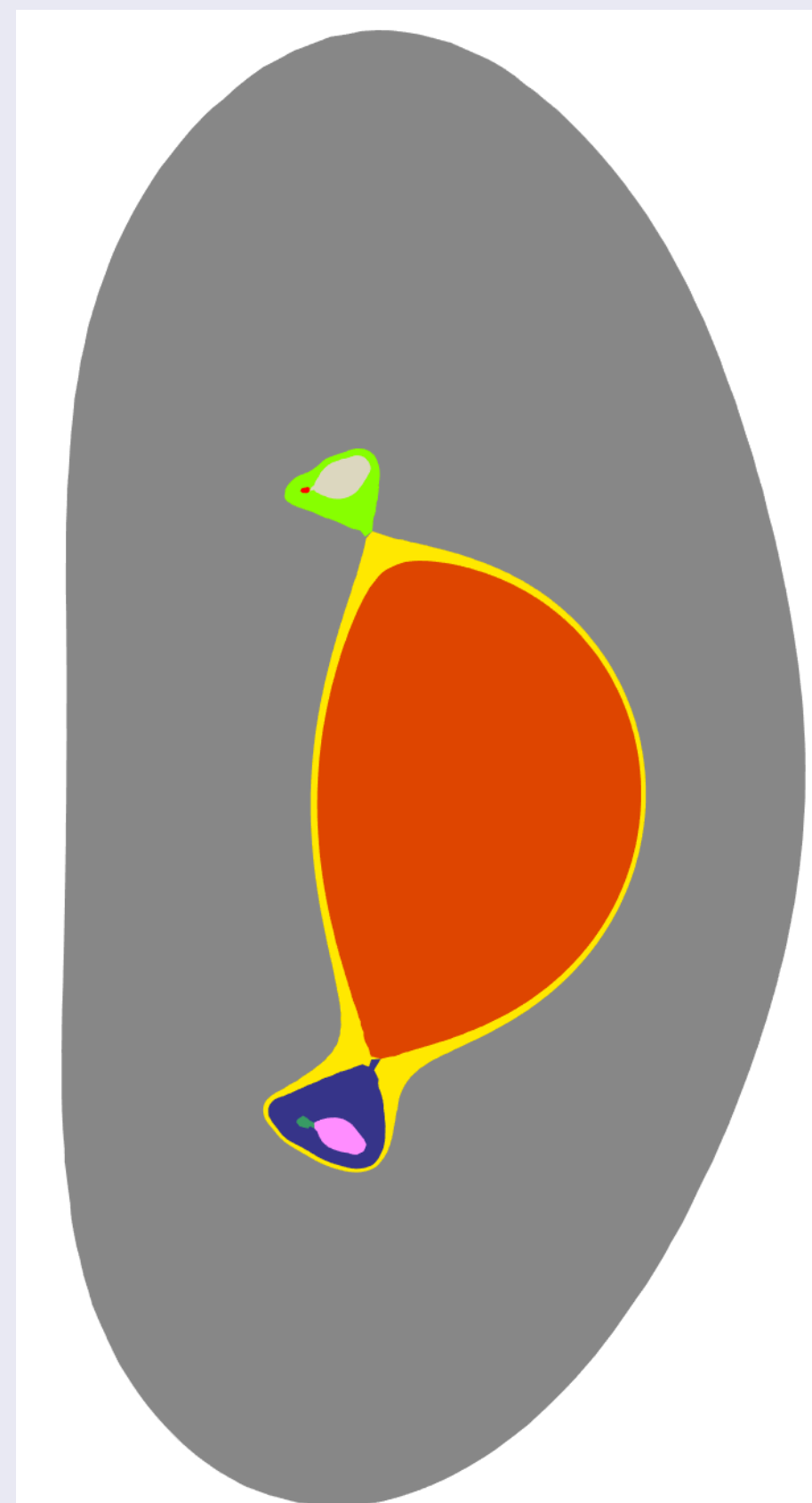
- Circle cells which are homeomorphic to open disks
- Circle bands which are homeomorphic to open annulus

## The Reeb graph

- Compute the connected components by computing the Reeb Graph



- Obtain the associated domain segmentation



each subdomain correspond to an edge of the Reeb graph and in each subdomain, the function is monotone

## Construction of the mapping of $\Omega_n$

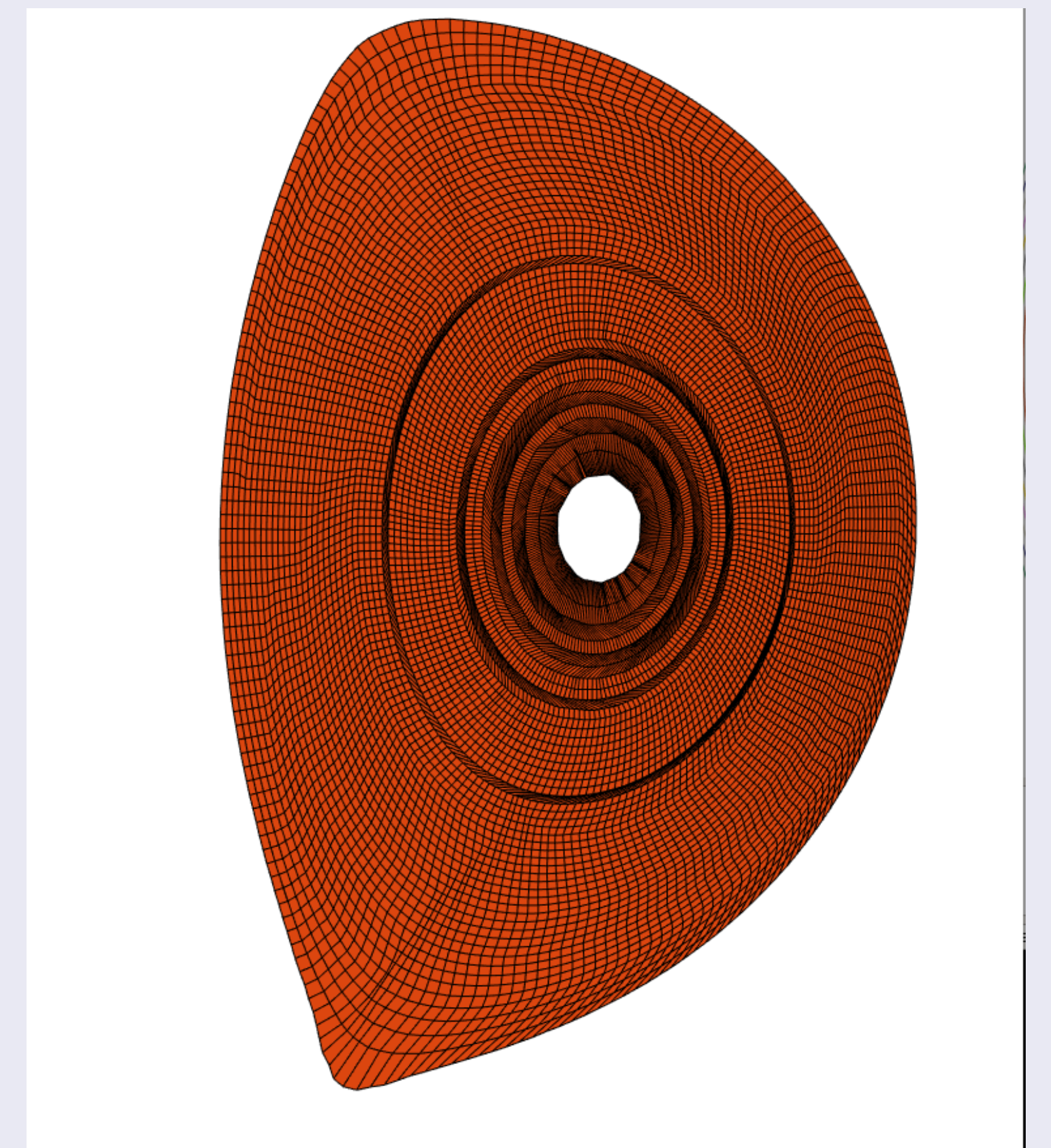
Orthogonal mesh constructed by intersecting

–The isolines

–Streamline integration of  $\frac{dx}{ds} = \nabla \psi$

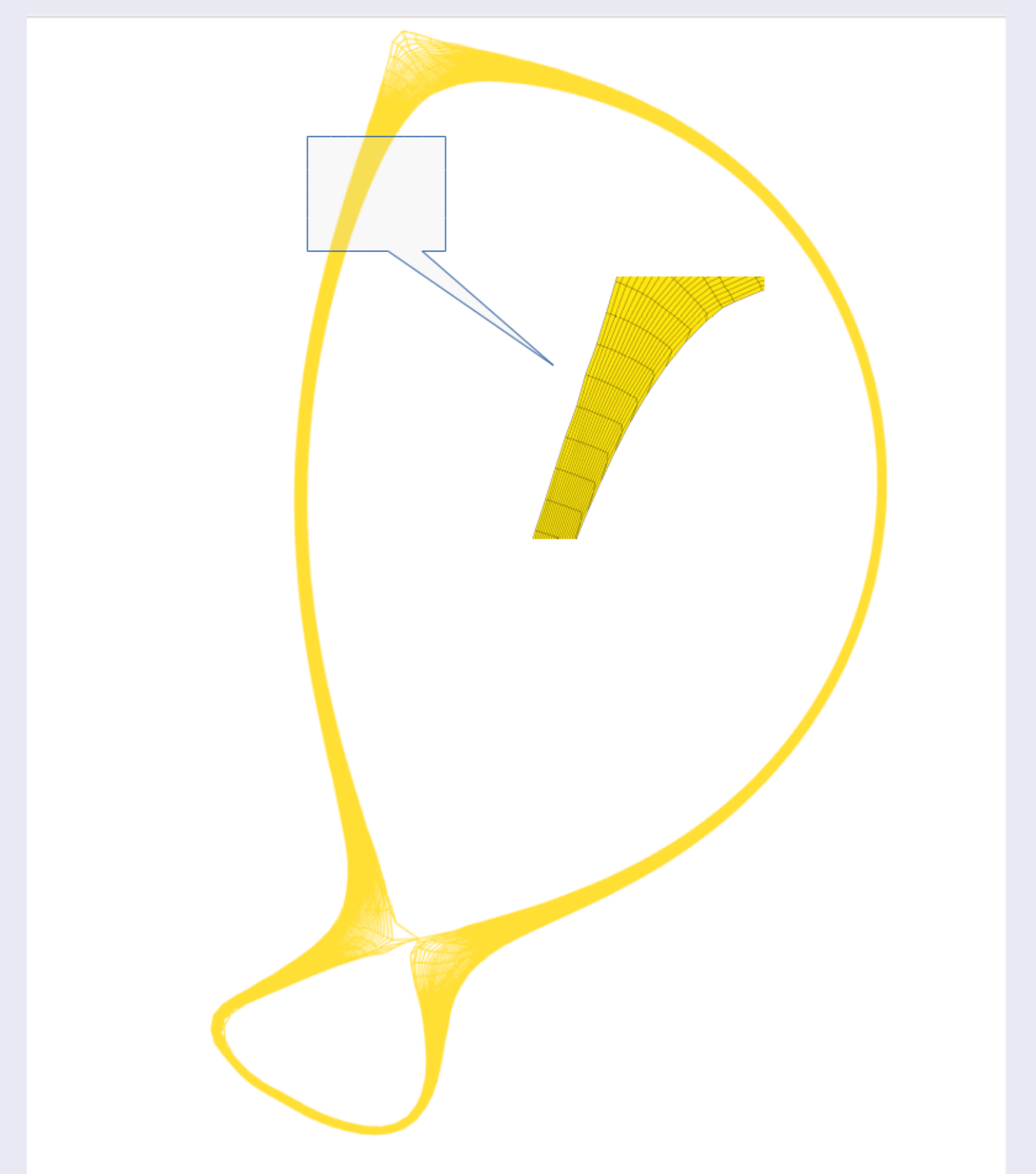
- Leaf of the Reeb Graph : homeomorphic to a disk

Example : Reeb Graph edge : [\[7925-8402\]](#)



- Internal edge of the Reeb graph : homeomorphic to annulus

Example : Reeb Graph edge : [\[7925-8402\]](#)



## Future works

- More accurate and smooth GS solvers
- Add additional mapping techniques : Elliptic solvers + equidistribution
- Topological simplification of the Reeb-Graph
- Construction of  $\mathcal{C}^1$  mappings
- Take into account the vacuum chamber boundary